INTERNATIONAL JOURNAL OF CRITICAL INFRASTRUCTURE PROTECTION 22 (2018) 100–112

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Criticality assessment of urban interdependent lifeline systems using a biased PageRank algorithm and a multilayer weighted directed network model

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ARTICLE INFO

Article history:
Received 21 July 2017
Revised 7 June 2018
Accepted 8 June 2018
Available online 14 June 2018

Keywords:
Lifeline system
Interdependency
Network model
Weighted directed network
Criticality assessment

ABSTRACT

Urban lifeline systems play vital roles in sustaining fundamental functionalities in urban areas. These systems, working collaboratively and synergistically, form a complex system of systems, in which disruptions in one system can rapidly propagate to others, posing a great challenge for the identification and protection of critical infrastructure facilities. This study introduces a new criticality assessment approach of interdependent lifeline systems. Given a weighted directed network of infrastructure systems, the proposed approach calculates vertex criticality through the biased PageRank algorithm: compared with the original PageRank algorithm, the biased one utilizes a personalization vector (in this context, vertex functional importance) in the process of criticality measurement. This algorithm design fulfils an integration of both network topology and function, and can comprehensively measure vertex criticality. To consider the impact of cascading failure, this criticality assessment method also adopts a linear combination form to take into account the criticality of child vertices using conditional probabilities as parameters. A case study is conducted on five real lifeline systems in a middle-sized county in China with over 300,000 inhabitants. Based on the case study model, targeted vertex attacks are carried out to illustrate the validity and effectiveness of this criticality measurement. Examining both network topological and function response, resulting curves show that the criticality ranking calculated with the proposed approach is better to reflect component topological and functional importance compared to other commonly used metrics. The main contribution of this study to the body of knowledge is the proposition of a new approach for criticality assessment of facilities in interdependent infrastructure systems under disaster scenarios, which provides a useful and intuitive guide for decision making process with regards to pre-disaster infrastructure protection.

1. Introduction

Critical Infrastructures (CIs) underpin every aspect of urban life by providing essential services [1,2]. According to the President's Commission on Critical Infrastructure Protection (PCCIP), CI is defined as 'a network of independent, mostly privately-owned, manmade systems and processes that function collaboratively and synergistically to produce and distribute a continuous flow of essential goods and services' [3]. Due to the rapid urbanization that has been witnessed worldwide in the past decades, urban systems are becom-

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https://doi.org/10.1016/j.ijcip.2018.06.002

Nomenclature

$G(V, E)$	the set of vertices and edges
V	the set of vertices
E	the set of edges
n	number of vertices
Q_{ij}	real flow from vertex V_i to V_j
W_i	weight (functional importance) of vertex V_i
W	weight vector of all vertices
d	damping factor
k_i	out degree of vertex V_i
$\overline{A}$	transmission matrix of the biased PageRank algorithm
$\bar{a}_{ij}$	elements of transmission matrix
PR_i	PR value of vertex V_i
$PR_i(t)$	t step iterative result for vertex V_i in the biased PageRank algorithm
$\theta_{i\|j}$	conditional probability of the failure of vertex V_i given the failure of vertex V_j
C_PR_i	cascading PR value of vertex V_i
d_{ij}	the length of the shortest path from vertex V_i to V_j
c_{ij}	flow capacity from vertex V_i to V_j
w_i	vertex capacity of vertex V_i
f_{ij}	Pseudo-flow from vertex V_i to V_j
u_i	waste of vertex V_i

ing ever-increasingly interdependent on each other, forming a large-scale complex system. These highly interconnected CIs show great vulnerability and sensitivity toward disruptions of their components [4], as the disruptions may propagate across CIs, which leads to cascading failures and causes functional breakdown of an entire city [5,6].

Cascading failures are not uncommon, and their destructive influence has been witnessed in numerous major disasters. For instance, the 2005 Hurricane Katrina caused severe destruction along the Gulf Coast of the United States. As a consequence of inadequate preparation for such catastrophic disruption to interdependent critical infrastructures and associated industries, the hurricane became 'the most destructive natural calamity in US history' [7], which destroyed the Gulf Coast's highways and oil supply infrastructure, flooded cities, and left an estimated three million people short of electricity supply. Similarly, the 2008 Chinese winter storm, raging throughout Southern China in mid-January, caused direct economic losses of 110 billion RMB and made 6.5 million people homeless for several weeks. The storm destroyed numerous transportation and electric power facilities in Southern China, and further disrupted countless interdependent infrastructures. Take Chenzhou, an economic and cultural center of Hunan Province, as an example. Due to cascading failures starting from the electric and water supplies that were directly paralyzed by the storm, people in this city experienced a complete cutoff of most public services including heating, telecommunications, public transportation, banking and finance for over ten days.

A critical job in urban disaster mitigation and prevention is to protect CIs to reduce potential hazard damage. Due to limited resources, infrastructure protection should be focused on facilities with high criticality. However, the inherent complexity among CIs poses great challenges for facility criticality measurement [8,9]. First, topology and function of CIs are so complex and not uniform that pure topological or functional metrics cannot fully represent the whole picture of infrastructure facility operation. Second, interdependent CI is regarded as a complex system, and thus problems within it cannot be separately studied on any single component [10]. The study presented in this paper proposes a new CI criticality assessment approach, featured by a biased PageRank algorithm given a multilayer weighted directed network as an input, for lifeline systems that have a technological network nature. This algorithm provides an effective and intuitive method, which emphasizes not only network topology but also network flow/performance, to evaluate infrastructure facility importance in maintaining network robustness after disruptions, and thus can be used to support the decision making in disaster mitigation practice.

The remainder of this paper is organized as follows. Section 2 reviews prior studies and discusses current research gaps, and outlines the proposed approach. Sections 3 and 4 provide details of criticality assessment and disruption simulation approach. Section 5 presents a case study of a middle-sized county in Southern China using the proposed approach. Section 6 summarizes findings, and discusses research limitations and future directions. All variables throughout this paper are summarized in the Nomenclature.

2. Literature review

The emergence of the research into infrastructure interdependencies is relatively recent [11,12], with the first known research systematically examining this concept published in 2001 [10]. Ever since, various modeling approaches have been explored to study interdependencies and to conduct resilience related analysis. Since many complex systems are really organized in the form of network structure, a major portion of these modeling approaches adopts network paradigm [13]. Compared with other modeling paradigms (e.g., agent-based modeling, system dynamics modeling, and input-output based modeling), network based models have advantages of being computationally inexpensive, intuitively explainable, and able to model different types (i.e., physical, cyber, geographical, and logical) of interdependencies [14–16].

With infrastructure systems encoded as networks, facilities/assets are represented mathematically as vertices while interactions between pairs of facilities/assets as edges [17]. For instance, vertices could represent substations in electric power system and water treatment plants in water supply system, while edges could represent transmission lines and pipelines respectively [18]. Based on network theory regarding network flow, optimization, and connectivity, network models have been built mainly for technical infrastructure systems, such as water distribution and electric power systems [6,12,19,20], internet and electric power systems [5], electric power and telecommunications systems [1,21], and gas pipeline and electric power systems [22].

Exploiting these network models, many researchers have analyzed how the networks respond in a relatively short period of time after different forms of attacks [21]. Some studies adopted random vertex removals [1,18,23,24]. Zhang et al. [25] pointed out that targeted attack causes more damage to interdependent networks than does random attack. Different from random one, targeted attack is simulated by removing vertices with largest degree, betweenness, centrality, and other topological metrics to simulate worst-case disruptions and examine network short-time response [1,6,26–31]. Among these studies, some conducted vertex removals on general networks to explore network properties from the perspective of graph theory and network science perspective. To mention two of them, Zhang et al. [30] examined the effect of interdependent network size on cascading failure and network robustness, under targeted attacks based on vertex betweenness on random scale-free networks; Huang et al. [24] analyzed Erdos–Renyi and scale-free networks and discovered that protection of highly connected vertices (i.e., those with high degrees) is beneficial to network robustness improvement. However, as Grubesic et al. [32] and Holme et al. [33] argued, few real CIs bear resemblance to the theoretical and general network models, suggesting these models cannot fully represent properties, behaviors, and functions of real-world interdependent CIs. There are also many studies focusing on a series of problems called Critical Node Problem (CNP), which goal is to find a set of vertices in the graph whose removals result in the largest damage to network topology (i.e., pairwise connectivity) [31,34,35]. These studies looked into undirected and unweighted graphs, and are often widely applied into social network research and vaccination research, which emphasize the existence more than the strength of relations between vertices.

In the context of CI, the order of targeted attacks (worst-case simulation) is usually determined by component criticality: the more critical a certain component is, the more severe impact its failure is to the whole network [21]. Identification of critical components is an important part of CI resilience/vulnerability analysis, and scholars have proposed various methods to assess facility criticality [32,36,37]. Some researchers assessed infrastructure facility criticality using site investigations, interviews and questionnaires with industry experts [2,8], though manual observations and expert opinions may be subject to personal experience and judgment compared to numerical evidence. Similar to those studies on artificial general networks mentioned above, many studies using real-world CI networks also adopt a pure topological perspective, from which system performance can only be referred as edge's existences [16], and commodity flows between pairs of vertices and across CIs are not considered [38]. For instance, critical components are usually identified by vertex degrees [26,39], characteristic path length [28], and centrality [40]. In addition to numerical metrics, an enumeration algorithm and a genetic algorithm were also proposed to identify critical vertices based on pure network topology [29,41]. Pinnaka et al. [42] compared four topological metrics, namely degree, closeness, betweenness and eigenvector centrality on US CI network, and found out that all these metrics based attacks have very similar impact on the network. In fact, unlike general networks, real-world CI networks consist of topology of vertices and edges, as well as engineering functionality such as flow and capacity [43], both of which are equally essential when assessing criticality [8]. There are a portion of studies using flow based metrics to identify critical components [5,37,44,45]: for example, Sullivan et al. [46] proposed travel time to identify critical road segments in transportation system; Matisziw and Murray [47] focused on the availability and operation of source-sink flow after disruptions and proposed an optimization model to identify infrastructure critical to system flow; Nicholson et al.[48] emphasized performance-driven measures of component importance, and proposed weighted flow capacity rate importance measurement that accounts for both flow and capacity of edges. However, there is no prior study that integrated topology and flow using one single criticality measurement approach. Although several studies [1,28] considered both network topology and function into their analysis, these two aspects were modeled differently and separately, which would lead to two separately calculated and somewhat inconsistent criticality indices.

This study proposes a method that integrates topology and function of a network in infrastructure facility criticality assessment. The study presented in this paper focuses on five urban lifeline systems: water supply, electric power, natural gas and oil, transportation, and telecommunications [49,50]. From a network perspective, these five CIs share a typical technological network structure, which are defined as 'man-made networks designed typically for distribution of some commodity or resource, such as electricity or information' [18]. Following a network based modeling paradigm for these technological networks, this paper proposes a new vertex criticality assessment implemented and illustrated in a multilayer weighted directed network model. Concretely, a biased PageRank algorithm is introduced and refined to calculate vertex criticality, which combines both network topology, embedded in adjacency relationship, and network function, embedded in network flow. To consider the impact of cascading failure, this criticality assessment method also adopts a linear combination form to take into account child vertices' criticality using conditional probabilities as parameters. A case study on five real lifeline systems in China is conducted to illustrate and validate the criticality assessment approach. Numerical results of vertex criticality rankings prove the effectiveness of this approach on both network topology and function, compared with traditional criticality metrics. Based on this approach, numerical criticality values and rankings of infrastructure facilities can be efficiently computed, which are believed to be useful for decision makers for better infrastructure protection and disaster mitigation before disruptive events.

3. CI criticality assessment approach

3.1. *A multilayer network model for urban interdependent lifeline systems*

The study takes a multilayer network model as input to conduct criticality assessment and related analysis. This input

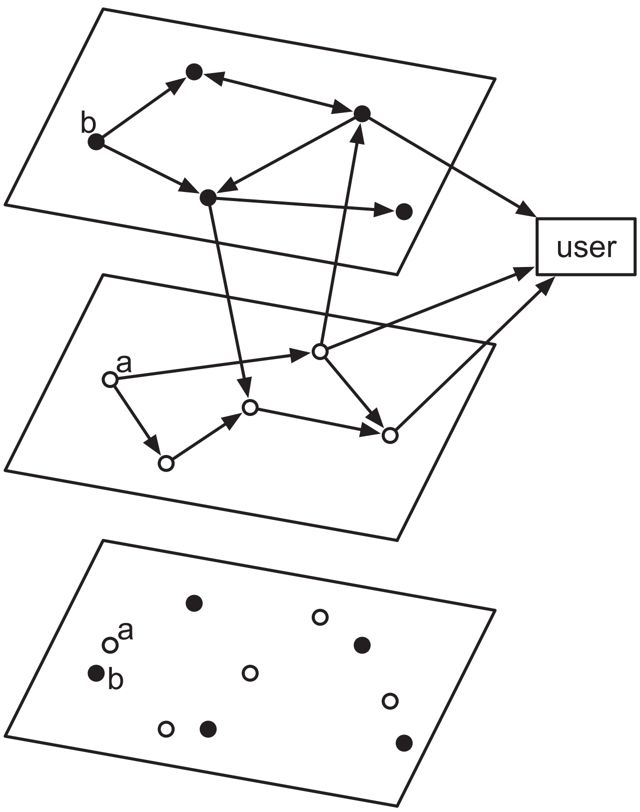


Fig. 1 – A simple three-layer network model of a fictional city case.

multilayer network is illustrated using a simple fictional case as shown in Fig. 1. There are two layers representing two lifeline systems connected by inter-edges (solid arrows in the figure), which corresponds to physical and cyber interdependencies under the classification proposed by Rinaldi et al. [10]. End users are modeled as a dummy sink vertex connected to all 'terminal' vertices that are able to provide service directly to users. For instance, in water supply system, reservoir vertex cannot provide drinking water to users directly and thus does not connect to end users.

Unweighted undirected networks, which can be considered as special cases of networks, have been used in most prior studies [15,28,51–53] to examine infrastructure topologies and relevant vulnerabilities from the topological perspective [45]. However, it is argued that the more general form of network, namely weighted directed network, is able to reflect more properties of urban lifeline systems. In a weighted directed network, vertices are assigned weights to represent their functional importance on transmitting commodity, and thus can be distinguished with other vertices even in the same layer; edges are assigned directions, and thus can distinguish supporting and supported relationship for a pair of vertices. Unlike existing studies in which flow properties are isolated from other properties (such as topology) in the models [54], the adopted network structure reflects both topological and functional properties by edge direction and weights.

The weighted directed network is formally defined as a graph $G = (V, E)$, where V is the set of weighted vertices ($|V| = n$) and E is the set of directed edges. To assist data collection and calculation for criticality assessment, this study introduces a new matrix termed flow matrix (FM) denoted as Q. Similar to graph adjacency matrix, FM is a square matrix ($Q \in \mathbb{R}^{n*n}$). Different from adjacency matrix with only binary elements, each element in FM Q_{ij} ($Q_{ij} \geq 0$) represents the real-time or average flow of the edge from vertex V_i to V_j. In particular, $Q_{ij} = 0$ means there is no commodity flow from V_i to V_j, either because they are not directly connected or because one of the two vertices is down. Note that flows can be in very different magnitudes and forms in the multilayer network model involving several lifeline systems. It is thus essential to normalize flows of edges so that dimensionless calculation across systems can be conducted. For each system, a min-max normalization is performed [55]. Min-max normalization maps the original flow value x to a [0,1] range. Specifically, if the minimum flow of all edges in a certain system layer is denoted as min, and the maximum flow of all edges in this system layer is denoted as max, then the flow value x of a give edge can be normalized as x′ based on the equation below:

$$x' := \frac{x - \min}{\max - \min} \quad (1)$$

After this normalization, all flows are converted to positive and dimensionless values within [0,1], which is consistent across systems and computable by the biased PageRank algorithm.

Based on FM, flow of a certain vertex is defined as the summation of all flow through it (both inbound and outbound edges), which can reflect to what extent the vertex contributes to commodity transmission in the network. Therefore, vertex functional importance (i.e. vertex weight) is defined as the proportion of its flow in the total flow of all vertices:

$$W_k = \frac{\sum_{i=1}^{n} Q_{ik} + \sum_{j=1}^{n} Q_{kj}}{2\sum_{i,j=1}^{n} Q_{ij}} \quad (2)$$

where W_k is the functional importance of vertex k. Since each edge is considered two times (both as inbound and as outbound edge) in the numerator, total flow in the denominator is multiplied by 2. Note that here W_k is normalized, since $\sum_k W_k = 1$. Vertex functional importance reflects how critical the vertex is to the CI engineering function. For a functionally important vertex, a malfunction or removal of this vertex and the resulting loss of its inbound and outbound edges would lead to significant loss of the overall functional performance of the network.

3.2. *A criticality assessment approach based on the biased PageRank algorithm*

This paper proposes a criticality assessment method based on PageRank (PR) algorithm. The original PR algorithm was designed by Brin and Page [56] for measuring the importance of numerous website pages for Google search. The core idea of the original PR algorithm is that more important website pages are more likely to be visited through hyperlinks from

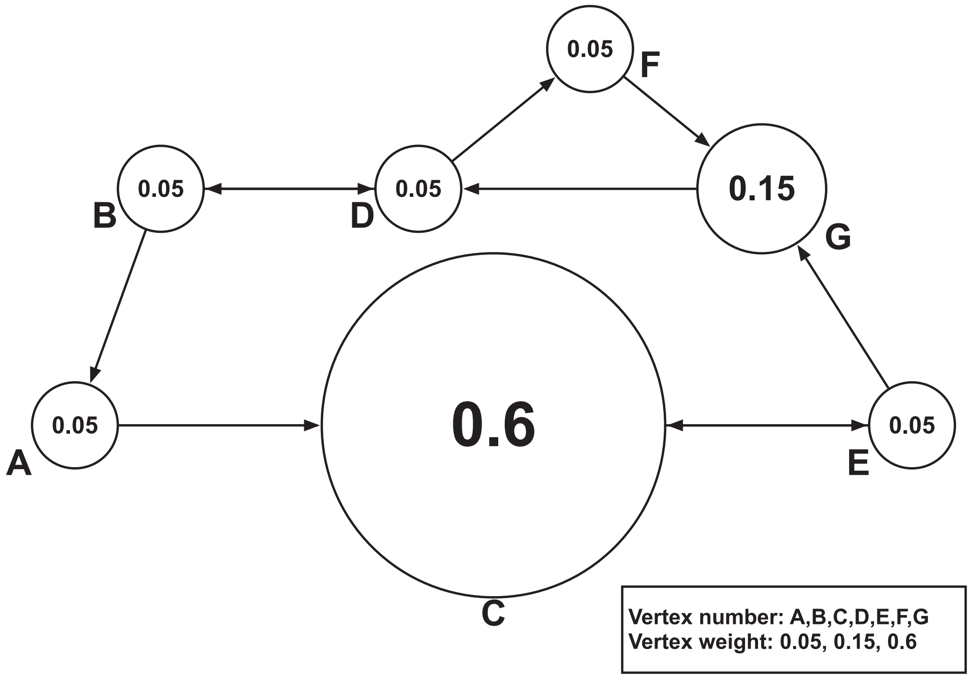


Fig. 2 – An example of a weighted directed network.

other pages. The algorithm assigns each page a value called the PR value, which is the probability that at a particular time a random web user is at this page. In each iteration of the original PR algorithm, the PR value of a given vertex is divided equally among and transmitted to all its child vertices. The PR value of each vertex at the beginning of the next iteration is the summation of PR values received from all its parent vertices. The iterative process terminates when all vertices' PR values fluctuate within an assumed small range (also called convergence) between iterations. This is in essence a Markov process, where vertices and stochastic transition matrix codetermine each iteration state [57].

In the context of infrastructure criticality assessment, this study adopts two adjustments to the original PR algorithm as described above. First, in the original PR algorithm, PR values at convergence reflect purely topological importance of the vertices, as all child vertices get the same share of PR values from their parent vertex. Among the variations of the original PR algorithm, there is a biased PR algorithm using a personalization vector, as a probability distribution to direct PR value transition in each iteration, to refine the original algorithm and to emphasize certain kinds of website pages, such as highly concerning topics [58]. The biased PR algorithm provides a promising solution for assessing the level of vertex criticality accounting for both topological and functional properties. Second, infrastructure network generally has at least one sink vertex (i.e., vertex without outbound edges), while these sink vertices will 'absorb' all PR values of vertices they are connected to and thus lead to failure of the algorithm [59]. To deal with this problem, this algorithm proposed here adopts damping factor with value of 0.85 as recommended in [56]. This damping factor works as a probability that a random walk on the network restarts at each iteration and thus offsets the importance of sink vertices in the network.

The details of the biased PR algorithm are explained as follows.

Step 1: Define a matrix $\bar{A}$ with its elements $\bar{a}_{ij}$ determined by :

$$\bar{a}_{ij} = \begin{cases} \dfrac{1}{k_i}, & \text{if there is flow from } i \text{ to } j \text{ in } Q \\ 0, & \text{if there is no flow from } i \text{ to } j \text{ in } Q \end{cases} \tag{3}$$

where k_i is the out-degree of vertex V_i. The matrix determines how PR value will be divided equally among child vertices for all vertices.

Step 2: Set random initial PR values $PR_i(0)$ $(i = 1, 2, \ldots, n)$ to all vertices, provided $\sum_{i=1}^{n} PR_i(0) = 1$, set an iteration count $t = 0$, and set a damping factor d at 0.85. Note that the selection of initial PR values does not influence the convergence results [60].

Step 3: Update the PR value of each vertex $PR_i(t)$ based on the following equation:

$$PR_i(t) = d \times \sum_{j=1}^{n} \bar{a}_{ji} PR_j(t-1) + (1-d) \times W \tag{4}$$

where $W = (W_1, W_2, \ldots, W_n)^T$, acting as the personalization vector with each element being vertex functional importance. This equation basically states that the updated PR value of each vertex is the sum of all PR values proportionally distributed by its parent vertices multiplied by d, plus a portion of the normalized functional importance multiplied by $(1-d)$.

Step 4: Repeat Step 3 until all PR_i's converge.

The advantage of the biased PR algorithm over the original PR algorithm is demonstrated using an example network shown in Fig. 2. This network is composed of seven vertices

Table 1 – Results of vertex criticality calculated by the original and biased PR algorithms for the example network in Fig. 2.

Vertex No.	Original PR	Biased PR
A	0.0787	0.0979
B	0.1956	0.1775
C	0.1349	0.2127
D	0.2524	0.2043
E	0.1334	0.1444
F	0.0761	0.0540
G	0.1287	0.1093

(Vertex A-G) representing seven users in a social network site such as Twitter and Weibo, and edges representing the relationship of following and being followed. Vertex size is approximately proportional to the vertex weight (i.e., functional importance, W in Eq. (2)), which is also noted in the figure. While the functional importance can have different meanings in different scenarios, in this example it reflects the relative number of posts and comments a certain user makes per day on this social network site. If considering both topological and functional importance, the intuition is that User C is more critical (or active) than other users. However, because such level of involvement in the social interactions is not reflected in the topological structure, the original PR algorithm is not able to recognize User C's criticality. As shown in Table 1, the original PR algorithm, only considering topological properties, ranks User D as the most critical, since intuitively User D is the most connected vertex in the network, while the biased PR algorithm (using the steps explained earlier and with damping factor d set to be 0.85) takes both topological and functional properties into consideration and ranks User C as the most critical instead. In fact, the latter one is a more reasonable assessment given the significant contribution of User C to the activeness of the social network. The two sets of criticality ranking are distinct as can be seen in the table.

To take into account the impact of cascading failures, it is also necessary to add the potential risk of child vertices' failure to the PR value. Cascading PR value for a certain vertex is hence defined as a linear combination of its own PR value and its child vertices' PR values.

$$C\_PR_i = PR_i + \sum_{j,(i,j)\in E} \theta_{j|i} \times PR_j + \sum_{k,(i,j)\in E,(j,k)\in E} \theta_{k|j}\theta_{j|i} \times PR_k \quad (5)$$

where j's are child vertices and k's are grandchild vertices of i, and PR_i, PR_j, and PR_k are the PR values of i, j, and k respectively, calculated through the iteration steps defined earlier. Moreover, both θ_{ij}'s and θ_{jk}'s are predefined propagation parameters indicating the conditional probability of failure of child vertex given the failure of parent vertex. Concretely, $\theta_{j|i}$ indicates the probability of failure of j given the failure of i, while $\theta_{k|j}$ indicates the probability of failure of k given the failure of j; therefore, given the failure of i, the probability of the failure of k is $\theta_{k|j}\theta_{j|i}$ according to the chain rule of probability. Though the appropriate values of propagation parameters are out of scope of this paper, for infrastructure vertex i which have more externality (i.e., negative impact on child vertices due to failure of itself), larger $\theta_{j|i}$ is recommended, making the impact of cascading failure more obvious. The above form is called two-step cascading PR value. If considering more than two-step propagation, C_PR_i should be also added with i's grand grandchild vertices' PR values.

4. Disruption simulation based on the criticality assessment approach

The proposed criticality assessment based on the biased PR algorithm can be used for targeted attack simulation. Targeted attack is focused on vertex and the impact spreads from initially disrupted vertex to others through physical or cyber interdependencies. Typical targeted attack disruptions (both intentional and unintentional) include mechanical breakdown, operational mistakes, and small-range man-made damages such as small-scale fires. In this study, targeted attack is simulated by removing disrupted vertex one by one in a descending order of vertex cascading PR values, in order to examine network topological and functional response to the worst possible nodal disruptions; and it is assumed that when a targeted attack occurs, the broken vertex loses its entire function. Note that except targeted vertex attack, there are many other forms of disruptions that might happen in urban areas. For example, for large-scale natural disasters and large-scale man-made damages, the initial impact is usually areal, which means that all vertices within this attacked region would be impacted [20,61]. Geographical region criticality involves comprehensive consideration of geographical properties and infrastructure layout, which is out of scope of this study.

Network two-aspect response includes topological and functional response, which are indicated by two metrics as follows. First, as an indicator of network topological response widely used in prior studies [22,26,28,62], characteristic path length (CPL) is used for measuring global efficiency of a network. After vertex removals, decrease in CPL is a measurement of damages in global efficiency of the whole network in fulfilling connectivity. CPL is defined as the harmonic mean of all geodesic paths between any pairs of vertices in the network, and can be calculated as:

$$CPL = \frac{1}{\frac{1}{2}n(n-1)} \sum \frac{1}{d_{ij}} \quad (6)$$

where n is the number of vertices, and d_{ij} is the length of the shortest path from V_i to V_j. It needs to be pointed out that compared with the common use of CPL for an undirected network, the shortest path between a pair of vertices is direction sensitive in the directed network, which means d_{ij} is not necessarily the same as d_{ji}. Second, considering that the fundamental role of CI is to provide and guarantee service to users especially after disruptions, maximum flow is used as the indicator of network functional response. Since service of different CIs is from different source vertex (e.g., electric power is from generator, and drinking water is from reservoir), maximum flow here refers to the maximum commodity flow from all vertices to serve the dummy user vertex defined earlier in the network structure, constrained by both vertex and edge capacity. In order to calculate maximum flow, it is necessary to first formally define related variables in the network. The capacity (or termed as supply) of vertex V_i, denoted by w_i, represents the maximum possible amount of flow this vertex can generate. In the context of interdependent infrastructure network, if the loss of flow in each vertex is negligible, only the dummy user vertex has $w_i < 0$. Moreover, the capacity of edge from vertex V_i to V_j, denoted by c_{ij}, represents the maximum possible amount of flow that can flow from V_i to V_j. Note that edge capacity may or may not be the same as Q_{ij} defined above, which is the actual flow from V_i to V_j and thus satisfies $Q_{ij} \le c_{ij}$. The pseudo-flow of edge from vertex V_i to V_j, denoted by f_{ij}, represents the net flow from V_i to V_j which cannot exceed the capacity c_{ij}. Note that pseudo-flow does not necessarily satisfy flow conservation, which requires the sum of inbound flows $\sum_j f_{ji}$ is equal to the sum of outbound flows $\sum_j f_{ij}$. Another parameter, the waste of vertex V_i, denoted as u_i, is defined as $\max\{0, \sum_j f_{ji} - \sum_j f_{ij} + w_i\}$. It refers to the 'wasted' portion of capacity of V_i not consumed by the current flow of commodities in the network. Based these notations, the maximum flow in this context can be converted to the Minimum-Waste Flow (MWF) problem of finding pseudo flows that minimize total waste in this network, and can be expressed mathematically as follows:

$$\begin{aligned}
&\text{Minimize} \quad \sum_{i \in V} u_i \\
&\text{subject to} \quad u_i + \sum_j f_{ij} - \sum_j f_{ji} \ge w_i \\
&\text{and} \quad 0 \le f_{ij} \le c_{ij} \\
&\text{and} \quad u_i \ge 0
\end{aligned} \tag{7}$$

where the first and third constraints are derived from the definition of vertex waste, and the second constraint from the definition of pseudo flow and edge capacity. This MWF problem can be easily solved by the Pseudo-flow Algorithm [63].

Note that for both network topological and functional response, this study considers short-time network response only (also known as the initial damage stage in infrastructure resilience process [64]), so dynamic redistribution, adjustment and other self-adapting mechanisms of both network topology and flow are considered out of scope and hence not modeled, which is consistent with various prior studies [1,27,28].

5. Case study

5.1. *Background of the case county*

A case study was carried out in a middle-sized county in Southern China to validate the proposed criticality assessment approach. Located at an intersection of several major economic regions in Southern China, the case county has an area of over 1000 km^2 and a population of over 300,000. The five lifeline systems of the county are well developed in recent decades. One lifeline system may involve several industries, for instance the telecommunications system involves multiple carriers and the transportation system involves roads and railways. For simplicity purpose, this case study selected the most data-accessible industry in each lifeline system. Table 2 lists the selected industries and vertices identified in these industries.

Table 2 – Selected industries in the five lifeline systems of the case study county.

Lifeline systems	Selected industry	Vertices
Electric power (E)	Electricity grids	Substations
Water supply (W)	Drinking water supplies	Reservoirs, water plants, pump stations
Natural gas and oil (N)	Gasoline and diesel supplies	Oil depots, gas stations
Transportation (T)	Roads	Road intersections
Telecommunications (TL)	Fiber-optic cables	Data centers

Table 3 – Basic statistics of the five lifeline system layers in the network model.

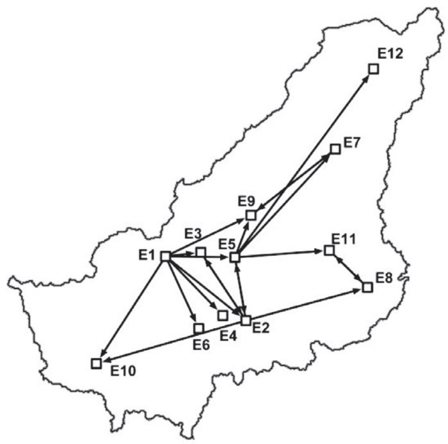
Lifeline systems	Number of vertices	Number of edges	Number of intra-level edges	Number of inter-level edges
E	12	87	21	66
W	19	20	20	0
N	15	14	14	0
T	32	107	43	64
TL	14	27	13	14

Data of the above lifeline systems, such as system basic functions, approximate locations, quantity and direction of commodity flows, were collected from responsible administrative agencies and companies for research purpose only. Certain types of data, such as exact accurate flow quantities of certain municipal services, were security or business sensitive and not available to the research team. Reasonable estimations were made after consulting experts from the administrative agencies and companies to ensure the completeness of data needed for modeling and analysis.

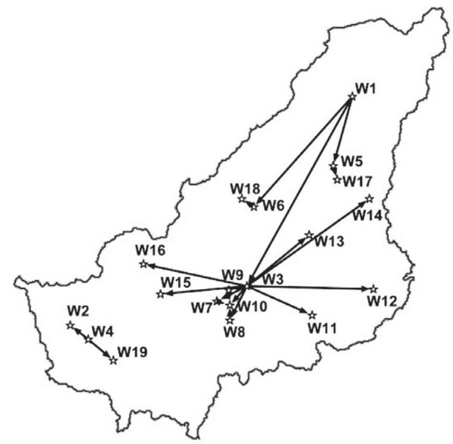
5.2. *Establishment of the network model*

After data collection, a network model was established for the five lifeline systems of the case county. The model consists of five system layers (weighted and bidirected), and basic statistics of the network is presented in Table 3.

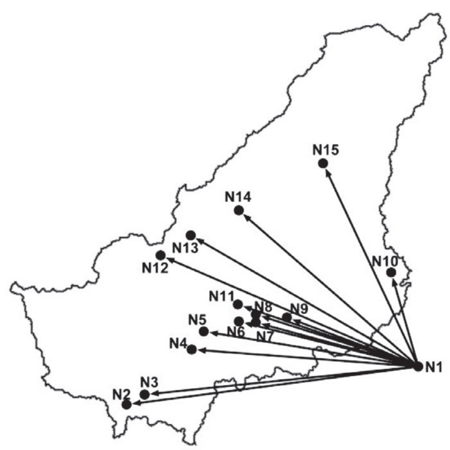
In layer E (Fig. 3a), the vertices represent one 220kv substation, five 110kv substations, and six 35kv substations. The 220kv substation receives high-voltage electricity from a nation-level electric power grid and distributes electricity to subordinate 110kv substations, serving as the sole source of electricity in the county. In layer W (Fig. 3b), the vertices represent two reservoirs, four water plants, three pump stations, and ten control valves. Water flows from the reservoirs downstream to the water plants; after treatment, the water flows through control valves for distribution to end-users. In layer N (Fig. 3c), the vertices represent one oil depot (outside the county boundaries) and fourteen gas stations. All of the gas



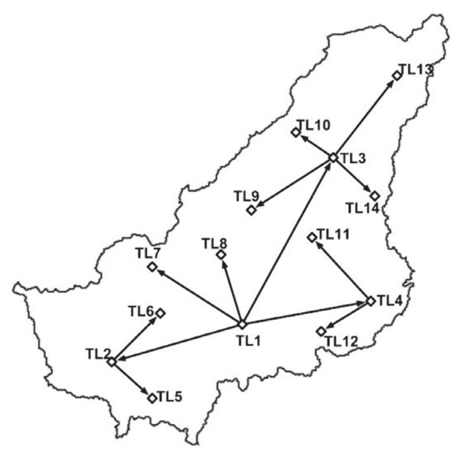
a. E layer



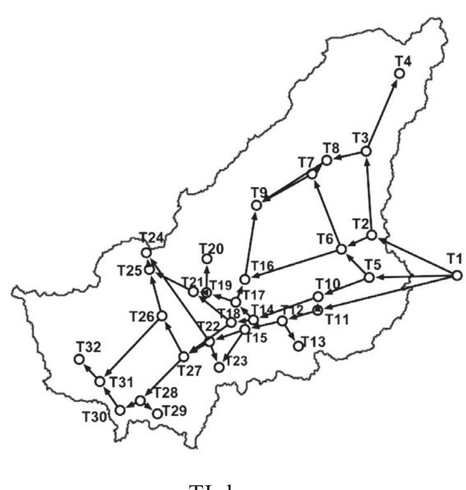
b. W layer



c. N layer



d. T layer



e. TL layer

Fig. 3 – Functional layers in the network model.

stations rely on oil supply from the oil depot via road tankers, and the fourteen intra-edges represent oil supply and demand between the oil depot and the gas stations. In layer T (Fig. 3d), the vertices represent 32 intersections on national highways, provincial roads and county-level roads. Unlike other lifeline systems, it is difficult to clearly identify the traffic source in the county. For convenience, a virtual vertex is thus set outside the county as a fictional source of the traffic. Traffic flows between pairs of intersection vertices are net flows of the two opposite directions. In layer TL (Fig. 3e), the vertices represent one first-level data center, three second-level data centers, and nine third-level data centers. Additionally, as illustrated in Fig. 1, a dummy user vertex, with its capacity set to be −10,000 (a sufficiently small negative number), is added here in this network model, representing the collection of end users.

The above layers are interdependent in a wide variety of ways. For instance, remote monitoring and telecommunications of reservoirs, water plants and pump stations in layer W rely on the telecommunication service provided by layer TL. The operation of water plants and pump stations requires electric power from substations in layer E. Discharge and transport of oil relies on road tankers that travel between the oil depot and gas stations. The operation and maintenance of gas stations require layer E, and remote monitoring of oil storage relies on layer TL. As for the transportation system, power or telecommunications outage would lead to the failure of traffic signals and would cause traffic congestion and interruption. Thus, each intersection requires service from layers E and TL. Telecommunications mainly rely on electric power. High-level data centers also require diesel supplies to run power generators and maintain electric power supply when regular power supply is disrupted. Note that in the case county, telecommunications between electricity substations are fulfilled via a separate system especially designed for electric power system, other than layer TL, due to security and stableness consideration. Therefore, there is no inter edge from layer TL to E. Fig. 4 presents a sketch of the multilayer network model of interdependent lifeline systems in the case county (at the bottom of the model, there is a geographical layer representing a common coordinate plane for the system layers above).

Based on the model, all vertices' criticality was calculated using the biased PR algorithm and considering two-step propagation with all propagation parameters (the conditional probabilities $\theta_{k|j}$ and $\theta_{j|i}$ in Eq. (5)) set to be 0.5. All the other parameters, such as damping factor and initial PR's, are set as initially designed in the biased PR algorithm.

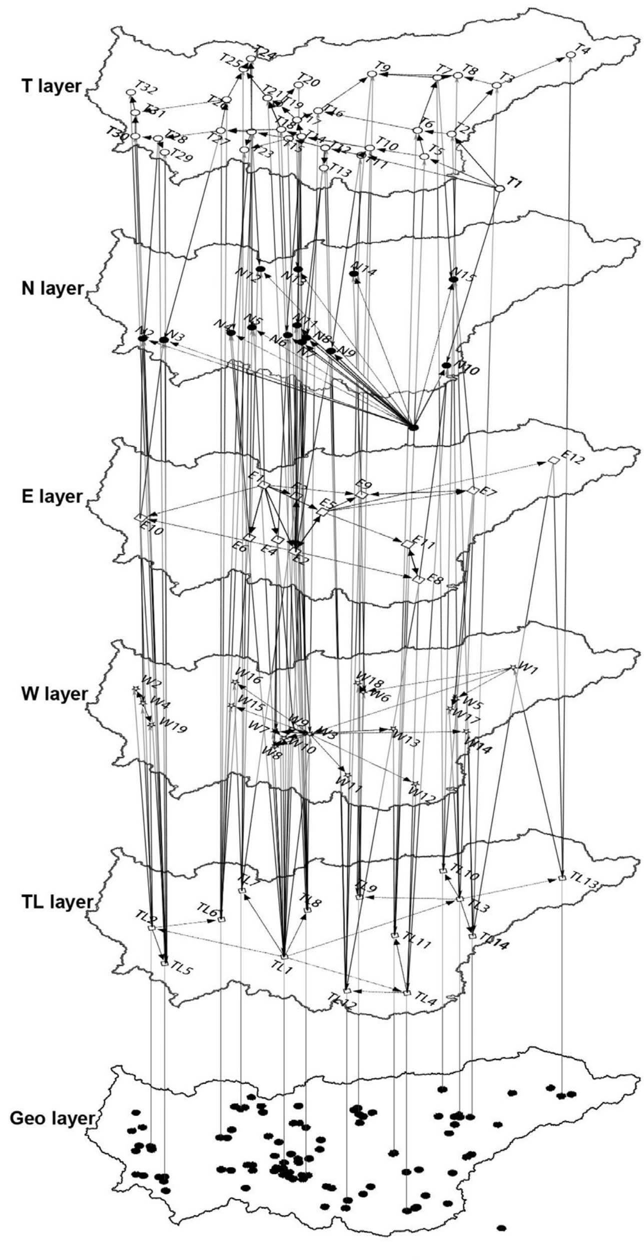


Fig. 4 – Multilayer network model of the case county.

5.3. *Vertex criticality assessment*

Based on the established network model, disruptions were simulated by strategically removing vertices, one at a time in the descending order of vertex criticality (i.e. cascading PR values). After each removal, two-step propagation was simulated, which means that the removed vertex's grandchild vertices also failed; and then the network was recovered back to the original state, in order to compare network response to each removal of single vertex. The results of network response measured in CPL and maximum flow are illustrated in Figs. 5a and 6a, respectively. To compare and validate the effectiveness of this criticality assessment approach, three traditional topology-based methods for unweighted directed network, namely vertex in-degree, out-degree and betweenness, and random vertex removal were also used, and the corresponding results are illustrated in Figs. 5b–e and 6b–e.

The results show that, compared to commonly used metrics and randomness, cascading PR value reflects a comprehensive vertex criticality considering both network topology and function. Concretely, for network function, as can be seen in Fig. 6b–d, there is no any expected trend of changes in maximum flow when using pure topological metrics, because the vertices that can make destructive damage to the network function were not ranked high in these three rankings.

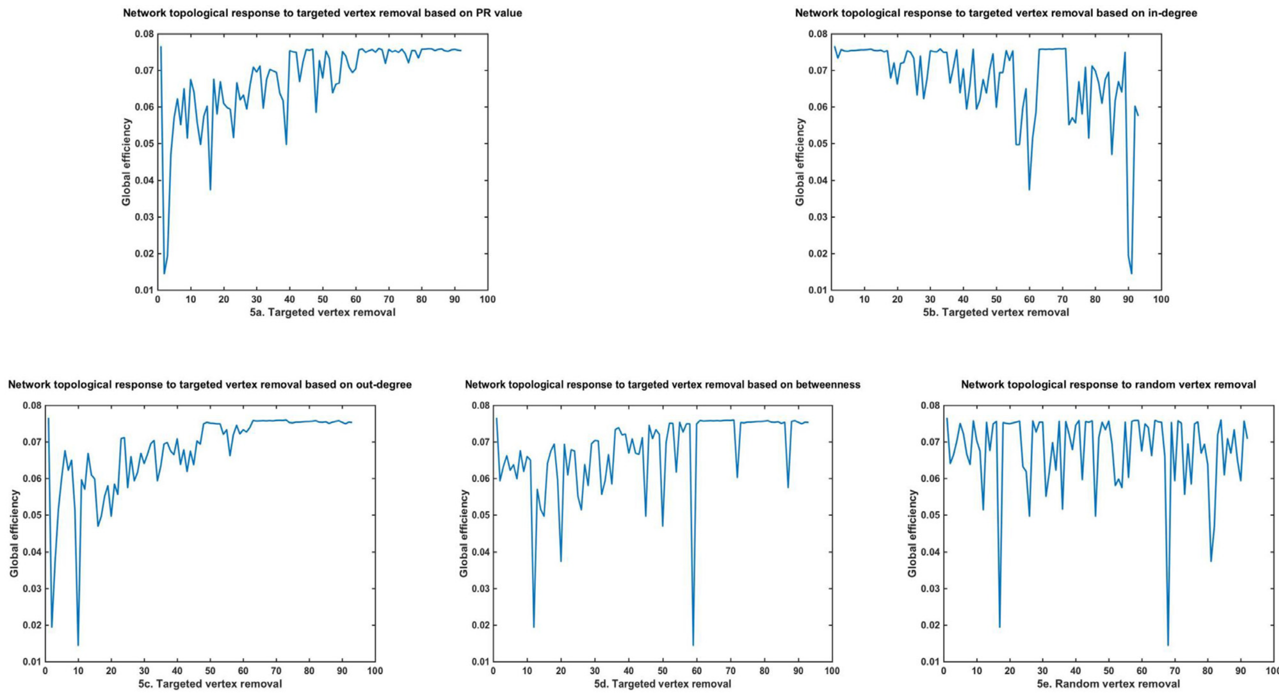


Fig. 5 – Changes of network topological response to targeted vertex removal (based on different vertex criticality measurements) and random vertex removal.

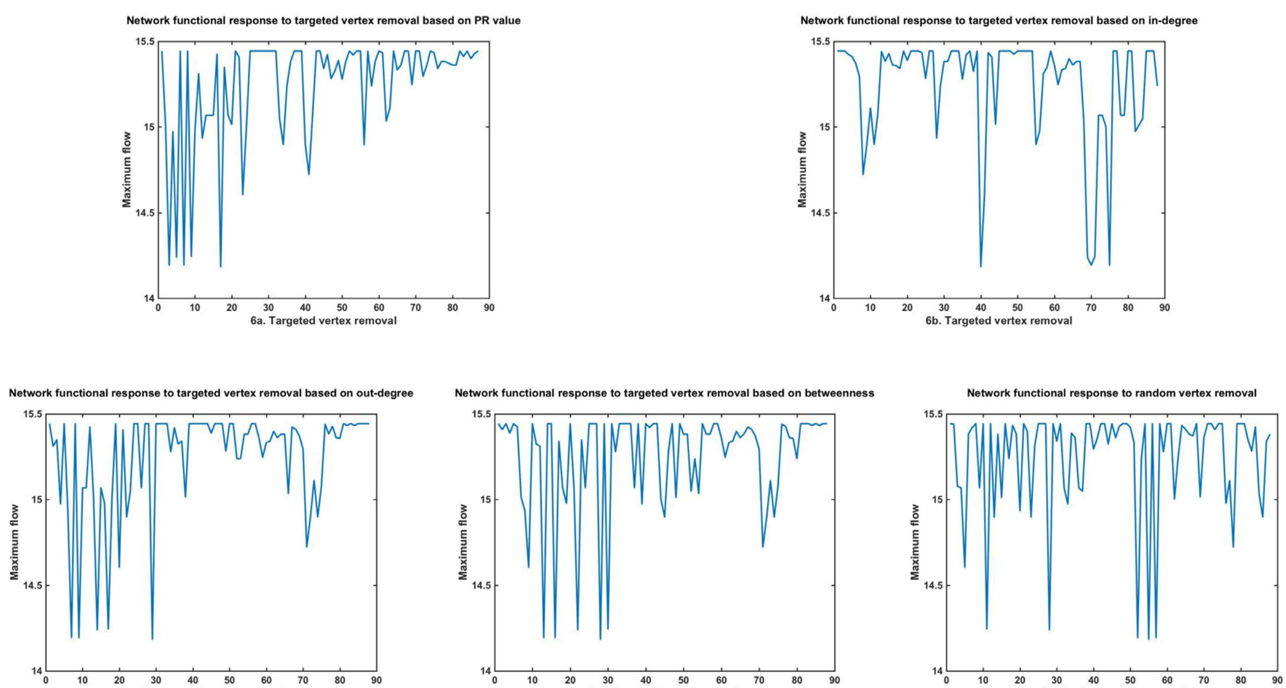


Fig. 6 – Changes of network functional response to targeted vertex removal (based on different vertex criticality measurements) and random vertex removal.

This proves what has been hypothesized before that topological metrics to a very large extent neglect the functional aspect. Only the curve of PR value based targeted attack has the expected pattern that as the removed vertex becomes less critical, the damage in the network flow that the removal caused becomes less obvious. On the other hand, for network topology, vertex in-degree and betweenness are worse than PR value and out-degree, while curves in Fig. 5a and c show a consistent and similar trend of increase in CPL as the removed vertex becomes less critical.

Actually, the fact that the out-degree based criticality measurement was found more reasonable than the other topology-based methods is not surprising, as out-degree is generally believed to contribute more to vertex criticality than in-degree and is therefore a better indicator of vertex topological importance. Examples supporting this argument include academic publications' citation network as explained by Wang et al. [59]. To judge the importance of a certain paper, its citation frequency (i.e. the number of papers getting support from this paper) is obviously a far more reliable indicator than its references (i.e. the number of papers giving support to the paper). Furthermore, a comparison of details in the PR ranking and the out-degree ranking reveals that the former keeps a greater balance between network topology and function. For instance, except the dummy user vertex, all source vertices (i.e. E1, N1, W1, T1 and TL1) were ranked as the most critical vertex in their respective system layer in the PR ranking. This is an appropriate reflection of the reality, because source vertices are all producers or sources of the commodities in their respective systems, and removals of these vertices would seriously damage the functionality of the system in transmitting the commodities. To take another practical instance, there was a large-scale water breakdown in this county, lasting for 58 hours starting from September 2nd, 2014. This incident, first recognized by a significant flow drop in a regulating valve, was due to mechanical failure in the outlet gate of reservoir W1, and led to an emergency state especially in the main city. The government deployed fire trucks to guarantee residential water use, and main factories utilized their backup water storage. Among the four ranking methods, only PR algorithm ranks W1, the practically critical vertex, higher than the other methods do. Nevertheless, vertices with many outbound edges but limited functional importance have higher rankings in the out-degree ranking than in the PR ranking. For instance, vertex E10 had 14 child vertices (i.e., W4, N1, N2, TL2, TL5, T22, T23, T26-32), and was ranked the 3rd in the entire network in the out-degree ranking. However, E10 accounted for only 3% of the total flow in the electric power system, and was ranked 66th in the PR ranking. Such discrepancies have been observed in many of other vertices, such as E9, E8, TL8 and E7, which proves that the PR ranking achieves a balance between topological and functional importance when assessing the criticality of the vertices.

6. Conclusion

Measuring the criticality of infrastructure facilities is fundamentally important for comprehensive resilience analysis of infrastructure systems [8]; however, there lacks research in the existing literature that uses an integrated approach to factor in both topological and functional aspects of an infrastructure network. The main contribution of this study to the body of knowledge is the proposition of a new approach for criticality assessment of facilities in interdependent infrastructure systems under disaster scenarios. Distinct from existing criticality assessments, this approach is based on a novel biased PageRank algorithm proposed in this study, which features the integrated consideration of both topological and functional attributes of infrastructure facilities modeled using a multi-layered complex network. The single numerical value (i.e., PR value) provides a better reflection of system operation and leads to a comprehensive identification of critical components in urban lifeline systems. To assess the resilience response of networked lifeline systems, urban disruptions can be simulated with vertex removals, based on vertex criticality ranking using the proposed approach. The changes of network global efficiency and maximum flow can be calculated as indicators of network topological and functional response to the simulated disruptions.

The proposed approach was applied in a case study conducted in five real lifeline systems of a middle-sized county in Southern China. The results demonstrated that the numerical criticality values well reflect both topological and functional importance in a unified form, thus proving the validity of the proposed approach. Though this case study built a simple and general network model for these lifeline systems without detailed and specific functionality, such as detailed flow pattern and physical rule modeling, this criticality assessment approach is not specially designed for certain types of network. As long as the input model embeds infrastructure system topology and function in edge direction and weight, the proposed approach can measure vertex criticality. Both criticality value and ranking are promising for practical use in urban disaster mitigation and prevention prior to disruptive events. As data sensing and information analysis are increasingly popular in the field of infrastructure building and management [65], there needs comprehensive and efficient quantitative approaches utilizing data to assist CI protection related decision making. As an interdisciplinary field, interdependent CIs pose a great challenge for effective and coordinative management, which highlights the vital importance of an intuitive and easy-to-understand criticality assessment [11].

While the approach has been proven promising, it also bears several limitations that require further investigation in future research. First, the determination of optimal value of conditional probability parameters for calculating cascading PR value (defined in Eq. (5)) under different failure scenarios has not been discussed in this study, and could be further investigated in future research. Second, considering model implementability and data accessibility, this study did not introduce domain-specific models with detailed functional design and flow patterns for each single system. However, integrating domain-specific models, such as the electrical power functional flow model proposed by Chopade and Bikdash [37], may better describe facility functions in various scenarios for different systems, and better reflect dependency relationships within each system. It may also better reflect real-world situations, and thus allow better accuracy and performance of the proposed criticality assessment approach. While such detailed modeling of the systems is out of scope of this study, it is a promising direction for future research. Third, this study focuses on technological networks, in particular lifeline systems, and considers social networks such as banking, education, and health care, as part of the external environment to the lifeline systems. Moreover, each lifeline system may contain several different but interdependent industries. How technological systems interact with the social networks, and how to reflect interdependencies within each lifeline system, should be further examined.

Acknowledgments

This material is based upon work supported by National Key Research and Development Program of China under Grant No. 2017YFC0803308, National Natural Science Foundation of China (NSFC) under Grants No. U1709212, 71741023 and 71741025, and Tsinghua University Initiative Scientific Research Program under Grants No. 2014z21050 and 2015THZ0. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the funding agencies. The authors are thankful for Prof. Feniosky Peña-Mora from Columbia University and Prof. Kincho Law from Stanford University for providing valuable advice on the preparation of this work.

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